Sample Question Paper - II

Mathematics - Class X

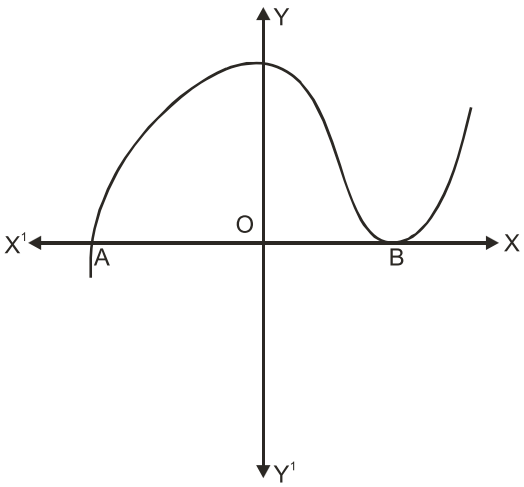
Time : Three hours **Max. Marks : 80**

General Instructions :

1. All questions are compulsory.
2. The question paper consists of thirty questions divided into 4 Section A,B,C and D. Section A comprises of ten questions of 01marks each, section B comprises of five questions of 02 marks each, section C comprises of ten questions of 03 marks each and section D comprises of five questions of 06 marks each.
3. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
4. There is no overall choice. However, internal choice has been provided in one question of 02 marks each, three questions of 03 marks each and two questions of 06 marks each. You have to attempt only one of the alternatives in all such questions.
5. In question on construction, drawings should be neat and exactly as per the given measurements.
6. Use of calculator is not permitted. However, you may ask for mathematical tables.

Section A

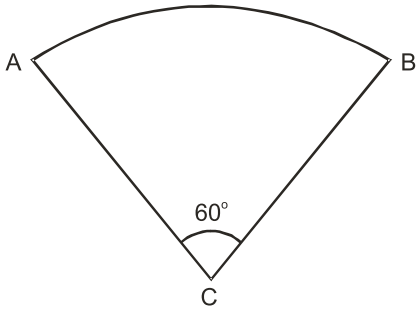
1. State the Fundamental Theorem of Arithmetic.
2. The graph of y=f(x) is given below. Find the number of zeroes of f(x).



3. Give an example of polynomials f(x), g(x), q(x), and r(x) satisfying $f(x) = g(x) \bullet q(x) + r(x)$ where deg r(x) = 0.

4. What is the nature of roots of the quadratic equation $4x^2 - 12x - 9 = 0$?

5. If the adjoining figure is a sector of a circle of radius 10.5 cm,



find the perimeter of the sector. (Take $\pi = \frac{22}{7}$)

6. The length of tangent from a point A at a distance of 5 cm from the centre of the circle is 4 cm. What will be the radius of the circle?

7. Which measure of central tendency is given by the x-coordinate of the point of intersection of the 'more than' ogive and 'less than' ogive?

8. A bag contains 5 red and 4 black balls. A ball is drawn at random from the bag. What is the probability of getting a black ball?

9. What is the distance between two parallel tangents of a circle of the radius 4 cm?

10. The height of a tower is 10m. Calculate the height of its shadow when Sun's altitude is 45°.

Section B

11. From your pocket money, you save Rs.1 on day 1, Rs. 2 on day 2, Rs. 3 on day 3 and so on. How much money will you save in the month of March 2008 ?

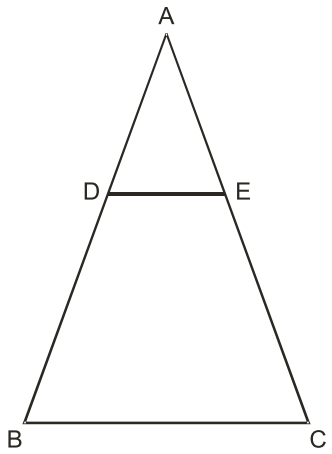
12. Express $\sin 67° + \cos 75°$ in terms of trigonometric ratios of angles between 0° and 45°

OR

If A,B,C are interior angles of a ΔABC, then show that

$$\text{Cos}\left(\frac{B+C}{2}\right)=\text{Sin}\frac{A}{2}$$

13. In the figure given below, DE // BC. If AD = 2.4 cm, DB = 3.6 cm and AC = 5 cm Find AE.



14. Find the values of x for which the distance between the point P (2,-3) and Q (x,5) is 10 units.

15. All cards of ace, jack and queen are removed from a deck of playing cards. One card is drawn at random from the remaining cards. find the probability that the card drawn is

    a) a face card

    b) not a face card

Section C

16. Find the zeroes of the quadratic polynomial $x^2 + 5x + 6$ and verify the relationship between the zeroes and the coefficients.

17. Prove that $5 + \sqrt{2}$ is irrational.

18. For what value or 'k' will the following pair of linear equations have infinitely many solutions

    $kx + 3y = k-3$
    $12x + ky = k$

OR

Solve for x and y

$$\frac{5}{x}+\frac{1}{y}=2$$

$$\frac{6}{x}-\frac{3}{y}=1$$

$\}x \neq o, y \neq o$

19. Determine an A.P. whose 3rd term is 16 and when 5th term is subtracted from 7th term, we get 12.

OR

Find the sum of all three digit numbers which leave the remainder 3 when divided by 5.

20. Prove that

$$\sqrt{\frac{\text{Sec A} - 1}{\text{Sec A} + 1}} + \sqrt{\frac{\text{Sec A} + 1}{\text{Sec A} - 1}} = 2\,\text{Cosec A}$$

21. Prove that the points A(-3,0), B(1,-3) and C(4,1) are the vertices of an isoscles right triangle.

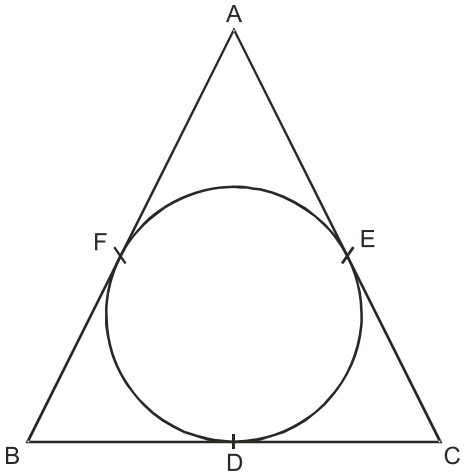
OR

For what value of 'K' the points A (1,5), B (K,1) and C (4,11) are collinear?

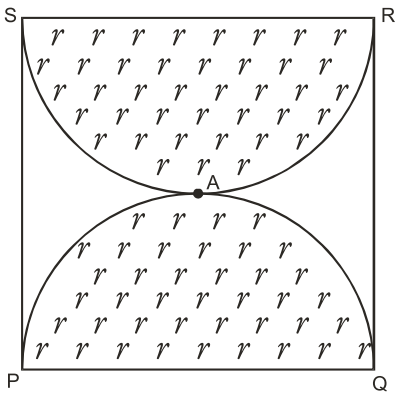
22. In what ratio does the point P(2,-5) divide the line segment joining A(-3,5) and B(4,-9)?

23. Construct a triangle similar to given ABC in which AB = 4 cm, BC = 6 cm and $\angle$ ABC = 60°, such that each side of the new triangle is ¾ of given Δ ABC.

24. The incircle of Δ ABC touches the sides BC, CA and AB at D,E, and F respectively. IF AB = AC, prove that BD=CD.



25. PQRS is a square land of side 28m. Two semicircular grass covered portions are to be made on two of its opposite sides as shown in the figure. How much area will be left uncovered? (Take $\pi = \frac{22}{7}$)



Section D

26. Solve the following system of linear equations graphically:

$$3x + y - 12 = 0$$
$$x - 3y + 6 = 0$$

Shade the region bounded by these lines and the x-axis. Also find the ratio of areas of triangles formed by given lines with x-axis and the y-axis.

27. There are two poles, one each on either bank of a river, just opposite to each other. One pole is 60m high. From the top of this pole, the angles of depression of the top and the foot of the other pole are 30° and 60° respectively. Find the width of the river and the height of the other pole.

28. Prove that the ratio of areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

Use the above theorem, in the following.

The areas of two similar triangles are 81 cm^2 and 144 cm^2. If the largest side of the smaller triangle is 27 cm, find the largest side of the larger triangle.

OR

Prove that in a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Use the above theorem, in the following.

If ABC is an equilateral triangle with AD $\perp$ BC, then $AD^2 = 3\,DC^2$.

29. An iron pillar has lower part in the form of a right circular cylinder and the upperpart in the form of a right circular cone. The radius of the base of each of the cone and cylinder is 8 cm. The cylindrical part is 240 cm high and the conical part is 36 cm high. Find the weight of the pillar if 1cm^3 of iron weighs 7.5 grams. (Take $\pi = \frac{22}{7}$)

OR

A container (open at the top) made up of a metal sheet is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm respectively. Find

(i) the cost of milk when it is completely filled with milk at the rate of Rs 15 per litre.

(ii) the cost of metal sheet used, if it costs Rs 5 per 100 cm^2

(Take $\pi = 3.14$)

30. The median of the following data is 20.75. Find the missing frequencies x and y, if the total frequency is 100.

Class Interval	Frequency
0 - 5	7
5 - 10	10
10 - 15	x
15 - 20	13
20 - 25	y
25 - 30	10
30 - 35	14
35 - 40	9

Marking Scheme

X Mathematics - Paper II

Section A

Q .No	Value Points	Marks
1.	Every Composite number can be factorised as a product of prime numbers. This factorisation is unique, apart from. the order in which the prime factors occur.	1
2.	Two	1
3.	One such example : $f(x) = x^2 + 1$, $g(x) = x + 1$, $q(x) = (x-1)$ and $r(x) = 2$	1
4.	Real and Unequal	1
5.	32cm.	1
6.	3cm	1
7.	Median.	1
8.	$\frac{4}{9}$	1
9.	8 cm	1
10.	10 m.	1
	Section B	
11.	Let money saved be Rs x	
	$\therefore$ x = 1+2+3+- - - +31 ($\because$31 days in march)	½
	$= \frac{31}{2}$ [1 + 31] $\quad \left[\because Sn = \left(\frac{n}{2}\right)(a+l)\right]$	½
	$= \frac{31}{\cancel{2}} \times \cancel{32}^{16}$	
	= 496	½
	Money Saved = Rs 496	½
12.	Sin 67° = Sin (90° - 23°)	½
	Cos 75° = Cos (90° - 15°)	½
	$\therefore$ Sin 67° + Cos 75°	
	= Sin (90° - 23°) + Cos (90° - 15°)	

Q .No	Value Points	Marks
	Cos 23° + SIn 15°	1
	OR	
	$\left(\begin{array}{l}\therefore A+B+C=180^{\circ} \\ \Rightarrow B+C=180^{\circ}-A\end{array}\right)$	½
	$\therefore \frac{B+C}{2}=90^{0}-\frac{A}{2}$	½
	$\therefore LHS=Cos\left(90^{0}-\frac{A}{2}\right)$	½
	$=Sin\ \frac{A}{2}$	½
	= R.H.S	
13.	In ABC, DE II BC,	
	$\therefore$ By B.P.T,	
	$\frac{AE}{EC}=\frac{AD}{DB}$	
	=> $\frac{AE}{AC-AE}=\frac{2.4}{3.6}=\frac{2}{3}$	1m
	= 3 AE = 2 (AC - AE)	
	= 5 AE = 2 AC	
	= 2 x 5cm	
	= AE = 2cm	1m
14.	Given PQ = 10 Units	
	$\therefore$ By Distance Formula	
	$\sqrt{(x-2)^{2}+(5+3)^{2}}=10$	
	$\Rightarrow$ $(x-2)^2 + 64 = 100$	1
	$\Rightarrow$ $(x-2)^2 = 36$	
	$\Rightarrow$ $x-2 = +6, -6$	½
	$\Rightarrow$ $x = 8, -4$	½

Q .No	Value Points	Marks
15.	Total Number of Cards = 52 Cards removed (all aces, jacks and queens) = 12 Cards Left = 52 - 12 = 40	½
	P (Event) = Total number of favourable outcomes / Total number of possible outcomes	½
	$\therefore$ P (getting a face Card) = $\frac{4}{40} = \frac{1}{10}$	½
	P (Not getting a face Card) = $1 - \frac{1}{10}$ = $\frac{9}{10}$	½
	Section C	
16.	$x^2 + 5x + 6 = (x+2)\ (x+3)$ Value of $x^2 + 5x + 6$ is zero When $x+2 = 0$ or $x+3 = 0$ i.e. $x = -2$ or $x = -3$	½ ½
	Sum of zeroes = (-2) +(-3) = - 5 = - $\left(\frac{5}{1}\right)$ = - $\left(\frac{\text{Co - efficient of x}}{\text{Coefficient of } x^2}\right)$	1
	Product of zeroes = (-2) x (-3) = 6	

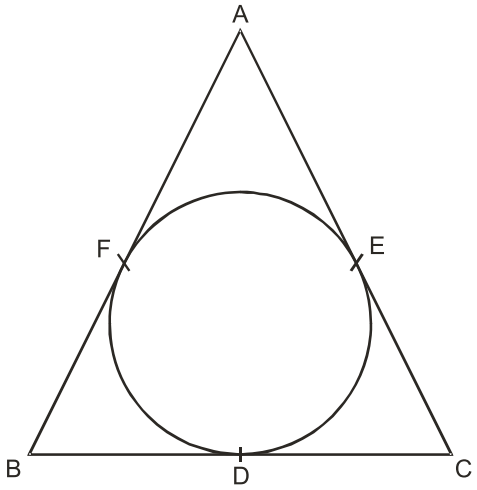
Q .No	Value Points	Marks
	$= \frac{6}{1}$	
	$= \left(\frac{\text{Constant term}}{\text{Coefficient of } x^2}\right)$	1
17.	Suppose $5 + \sqrt{2}$ is a rational number, say n.	½
	$\Rightarrow \sqrt{2} = n - 5$	
	As n is rational and we know that 5 is rational,	
	$\therefore$ n - 5 is a rational number.	
	$\therefore \sqrt{2}$ is a rational number	½
	Prove that $\sqrt{2}$ is not a rational number	1½
	$\therefore$ Our supposition is wrong	
	Hence $5 + \sqrt{2}$ is an irrational number	½
18.	For infinitely many solutions	
	$\frac{k}{12} = \frac{3}{k} = \frac{k-3}{k} \quad (k \neq 0)$	1
	$\frac{k}{12} = \frac{3}{k}$	
	$= k^2 = 36$	
	$= k = +6$	1
	$\frac{3}{k} = \frac{k-3}{k}$	
	$\Rightarrow 3 = K-3 \quad (k \neq 0)$	
	$\Rightarrow k = 6$	½
	The required value of k is 6.	½
	OR	
	Put $\frac{1}{x} = u$	
	$\frac{1}{y} = v$	

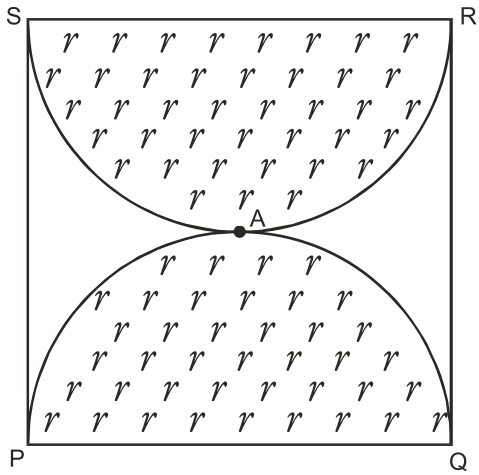
Q .No	Value Points	Marks
	$\therefore$ 5u + v = 2 (i) 6u -3v =1 (ii)	½
	Multiplying equation (i) by 3 and adding to (ii) we get 15u +3v = 6 6u - 3v = 1 Adding 21u = 7 $u = \frac{7^{1}}{21_{3}} = \frac{1}{3}$ $u = \frac{7^{1}}{21_{3}} = \frac{1}{3}$ From (i) v = 2 – 5u $= 2 - 5\left(\frac{1}{3}\right)$ $= \frac{6-5}{3}$ $v = \frac{1}{3}$ $\therefore$ x = 3 y = 3	½ ½ ½ 1
19.	Let the A.P be a,a+d, a+2d, - - - - a is the first term, d is the common difference It is given that a + 2d = 16 (1) (a+6d) - (a+4d) = 12 (2) From (2), $\cancel{a}$ + 6d - $\cancel{a}$ - 4d = 12 2d = 12 d = 6 Put d = 6 in (1) a = 16 - 2d = 16 - 2 (6)	½ ½ ½ ½

Q .No	Value Points	Marks
	$= 16 - 12$	
	$= 4$	½
	Required A.P. is 4,10,16,22- - - -	½
	OR	
	The three digit numbers which when divided by 5 leave the reminder 3 are	
	103, 108, 113, - - - - , 998	½
	Let their number be n, then	
	$t_n = a + (n-1)d$	
	$998 = 103 + (n-1)\,5$	½
	$= 103 + 5n - 5$	
	$5n = 998 - 98$	
	$n = \frac{\cancel{900}}{\cancel{5}}\ 180$	
	$n = 180$	1
	Now, $S_n = \frac{n}{2}\,[a + l]$	
	$S_{180} = \frac{180}{2}\,[103 + 998]$	½
	$= 90 \times 1101$	
	$= 99090$ Ans.	½
20.	L.H.S.	
	$= \sqrt{\frac{secA - 1}{secA + 1}} + \sqrt{\frac{secA + 1}{secA - 1}}$	½
	$= \frac{secA - \cancel{1} + secA + \cancel{1}}{\sqrt{sec^2A - 1}}$	1
	$= \frac{2\sec A}{\sqrt{\tan^2 A}}$ $(\because\ Sec^2 A - 1 = \tan^2 A)$	½

Q .No	Value Points	Marks
	$= \frac{2 \sec A}{\tan A}$	½
	$= 2 \operatorname{cosec} A$	
	= R.H.S.	½
21.	By distance formula	
	AB $= \sqrt{(1+3)^2 + (-3-0)^2}$	
	$= \sqrt{4^2 + (-3)^2}$	
	$= \sqrt{16+9}$	
	$= \sqrt{25}$	
	= 5 units	
	BC $= \sqrt{(4-1)^2 + (1+3)^2}$	
	$= \sqrt{3^2 + 4^2}$	
	$= \sqrt{25}$	
	= 5 units	
	AC $= \sqrt{(4+3)^2 + (1-0)^2}$	
	$= \sqrt{7^2 + 1^2}$	
	$= \sqrt{49+1} = \sqrt{50} = 5\sqrt{2}$ units	1
	Since AB = BC =5	
	Δ ABC is isosceles (1)	½
	Now, $(AB)^2 + (BC)^2$	
	$= 5^2 + 5^2$	
	= 25 + 25	
	= 50	
	$= (AC)^2$	
	$\therefore$ By converse of pythagoras theorem	

Q .No	Value Points	Marks
	Δ ABC is a right triangle (2)	1
	From (1) and (2)	
	Δ ABC is an isosceles right triangle	½
	OR	
	We have A (x_1, y_1) = A (1,5)	
	B (x_2, y_2) = B (K,I)	
	C (x_3, y_3) = C(4, 11)	
	Since the given points are collinear the area of the triangle formed by them must be 0.	1
	$[x_1 (y_2-y_3) + x_2 (y_3-y_1) + x_3 (y_1 - y_2)] = 0$	½
	=> 1 (1-11) + K (11-5) + 4 (5-1) = 0	½
	=> -10 + 6 K + 4 (4) = 0	
	=> 6K + 6 = 0	½
	=> 6K = - 6	
	K = -1	
	The required value of K = -1	½
22.	Let the point P(2, - 5) divide the line segment joining A(-3,5) and B (4,-9) in the ratio K : 1	½
	K : 1 B (4, - 9) A (-3, 5) P (2, -5)	
	By Section formula	
	$2 = \frac{4k-3}{k+1}$	1
	$\therefore$ 2(k+1) = 4k - 3 ½	

Q .No	Value Points	Marks
	$-2k = -5$	
	$k = \frac{5}{2}$	½
	$\therefore$ The required ratio is 5:2	½
23.	For constructing Δ ABC	1
	For constructing similar traingle to Δ ABC with given dimensions	2
24.		
	Since the lengths of tangents drawn from an external point to a circle are equal	½
	$\therefore$ we have	
	AF = AE - (1)	
	BF = BD - (2)	
	CD = CE - (3)	½
	Adding 1, 2 and 3, we get	
	AF + BF + CD = AE + BD + CE	1
	AB + CD = AC +BD	
	But AB = AC (given)	½
	$\therefore$ CD = BD	½

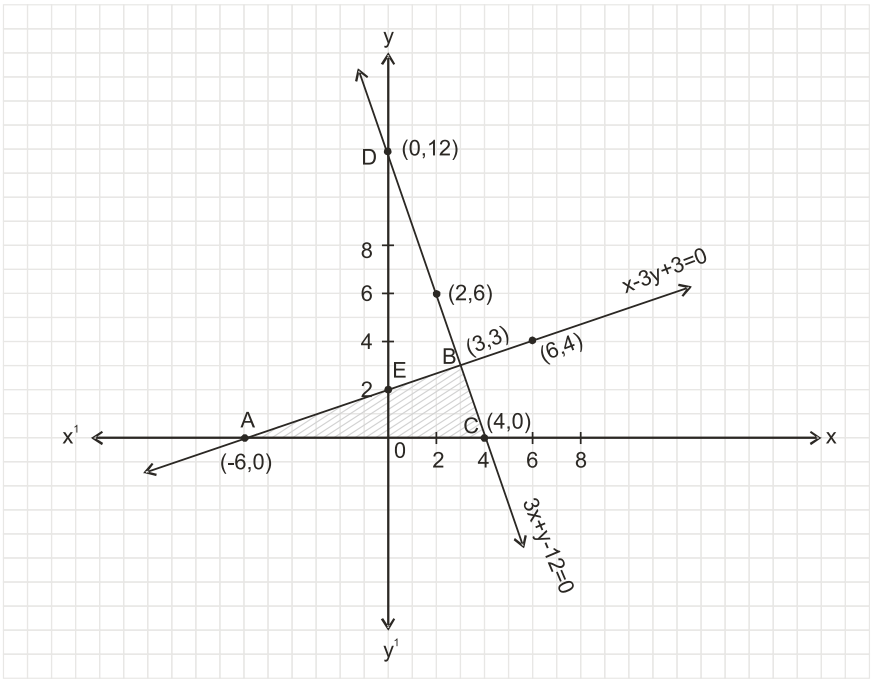
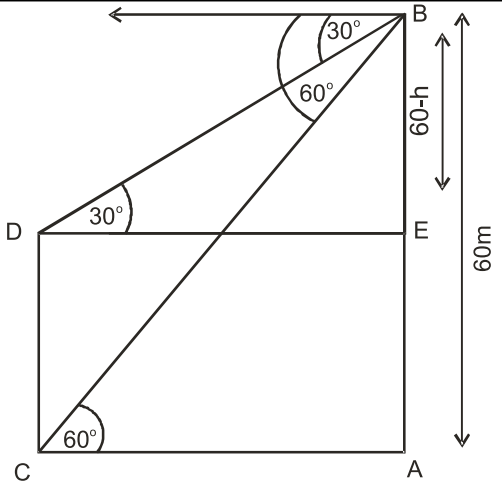
Q .No	Value Points	Marks
25.		
	Area left uncovered	
	= Area (Square PQRS) - 2 (Area of Semircircle PAQ)	1
	= $[(28 \times 28) - 2\frac{1}{2}(\frac{22}{7}(14)^2)]m^2$	1
	= $(784 - \frac{22}{7} \times 14 \times 14)\ m^2$	½
	= $(784 - 616)\ m^2$	
	= $168\ m^2$	½
Q.26	**Section D**	
	We have $3x + y - 12 = o$	
	$y = 12 - 3x$	

x	2	3	4
y	6	3	0

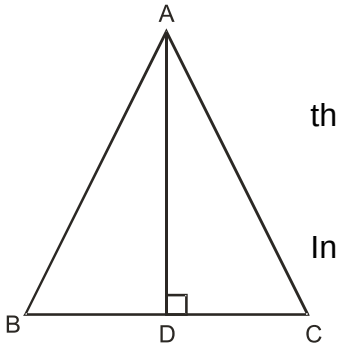
and $x - 3y + 6 = 0$

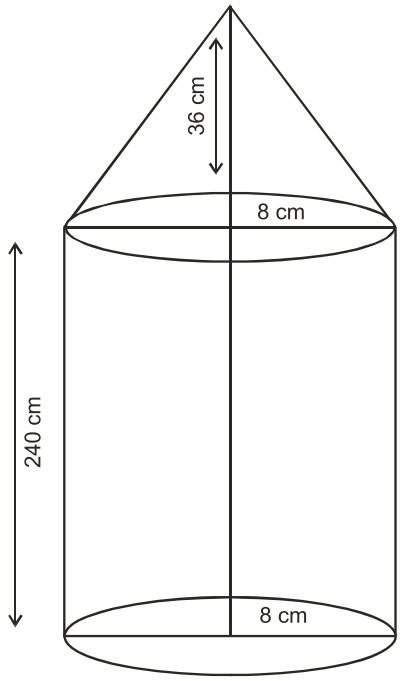
$$y = \frac{6 + x}{3}$$

x	3	6	-6
y	3	4	0

Q .No	Value Points	Marks
		2
	Since the lines intersect at (3, 3), there is a unique solution given by x=3, y = 3	1
	Correct shaded portion	½
	Area of triangle ABC formed by lines with x - axis = ½ x 10 x 3 = 15sq. units	1
	Area of triangle BDE formed by lines with y - a x is = ½ x 10 x 3 = 15 sq units	1
	∴ Ratio of these areas = 1 : 1	½
27.	 Correct figure	1

Q .No	Value Points	Marks
	Let AB be the first pole and CD be the other one.	
	CA is the width of the river.	
	Draw DE $\perp$ AB.	
	Let CD = h metre = AE	
	BE = (60-h) m	½
	In rt. (Δ BAC), $\frac{BA}{CA} = \tan 60^\circ$	½
	$\frac{60}{CA} = \sqrt{3}$	
	$CA = \frac{60}{\sqrt{3}}$	
	$= 20\sqrt{3}$	1
	$\therefore$ width of river $= 20\sqrt{3}$	
	or	
	= 34.6m	½
	Now, In rt. (Δ BED)	½
	$\frac{BE}{DE} = \tan 30^\circ$	
	$\therefore \frac{60-h}{20\sqrt{3}} = \frac{1}{\sqrt{3}}$	½
	60-h = 20	
	h = 40	1
	$\therefore$ Height of the other pole = 40m.	½

Q .No	Value Points		Marks
28.	Given, to prove, construction and figure	½ x 4	2
	Correct Proof		2
	Let the largest side of the larger triangle be x cm, then		
	$\frac{x^2}{27^2}=\frac{144}{81}$ (Using the theorem)		1
	$\therefore$ x = 36cm		1
	OR		
	Correct given, to prove, construction and figure	½ x 4	2
	Correct proof		2
	Let AC = a units		
	 then DC = $\frac{a}{2}$ units		½
	In rt Δ ADC, by the above theorem $AD^2 + DC^2 = AC^2$		½
	$AD^2 = a^2 - \left(\frac{a}{2}\right)^2 = a^2 - \frac{a^2}{4}$		
	$AD^2 = 3\left(\frac{a}{2}\right)^2 = 3DC^2$		
	$\therefore$ $AD^2 = 3DC^2$		1

Q .No	Value Points	Marks
29.		

Radius of base of Cylinder (r) = 8cm

Radius of base of Cone(r) = 8cm

Height of Cylinder (h) = 240cm

Height of Cone (H) = 36cm — 1

Total volume of the pillar

= Volume of cylinder + volume of Cone — ½

$= \pi r^2 h + \frac{1}{3} \pi r^2 H$ — 1

$= \pi r^2 (h + \frac{1}{3} H)$

$= \frac{22}{7} \times 8 \times 8 [240 + \frac{1}{3} (36)] \text{ cm}^3$

$= (\frac{22}{7} \times 8 \times 8 \times 252) \text{ cm}^3$

$= 50688 \text{ cm}^3$ — 2